

Recall that for a string w and $\sigma \in \Sigma$, $\#_{\sigma}(w)$ is the number of occurrences of σ in w .

- (1 mark) Let L be the set of strings over $\{a, b\}$ that contain the substring aab . What is the closure of L under concatenation?

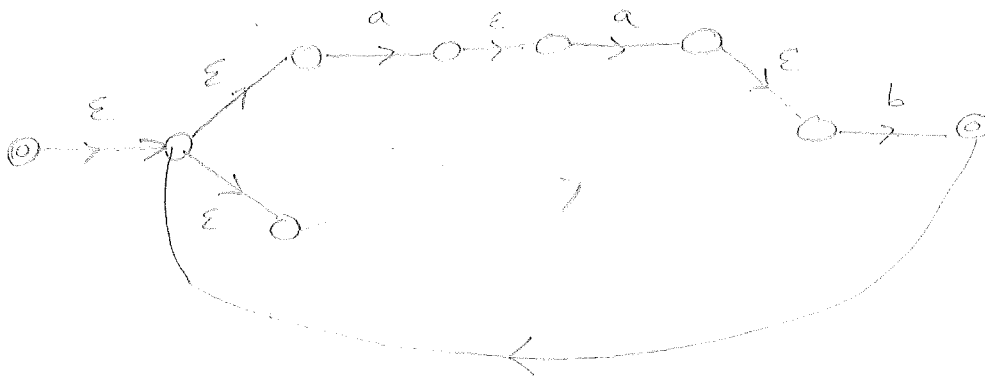
L

- (1 mark) What is the closure of the set $\{6, 8\}$ under the binary operator "average"? Note that $\text{average}(x, y) = \frac{x+y}{2}$.

$$\{x \in \mathbb{R} : 6 \leq x \leq 8\}$$



- (4 marks) Using the construction, give the NFA that corresponds to the regular expression $((aa + \epsilon)b)^*$. Do not include a dead state. Do include all ϵ transitions that the construction dictates (even if they are not useful).



- (10 marks) For each pair of regular expressions below, do they represent the same language? Answer True or False. If False, give a string that differentiates them (i.e., is in the language of one but not the other).

(a) $((a + \epsilon)(b + \epsilon))^*$ and $(a + b)^* + ab$

T

is in both
(b) $(a^*b)^+ aa(ba^*)$ and $(a + b)^* baab(a + b)^*$

F $baabbb$

(c) $(a^* + b^+)^*$ and $(a + b)^*$

T

(d) $(a + b)^* ab(a + b)^*$ and $a^*b(a + b)^*$

F b

(e) $(a^*b)^*$ and $(a + b)^*b$

F ϵ

- (3 marks) Give a regular expression for the language of non-empty strings over $\{a, b\}$ that contains no more than three b 's.

$$a(a^*ba^*ba^*ba^*)^*$$

$$a(a^*(\epsilon + b)a^*(\epsilon + b)a^*(\epsilon + b))a^* + b(a^*(\epsilon + b)a^*(\epsilon + b))a^*$$

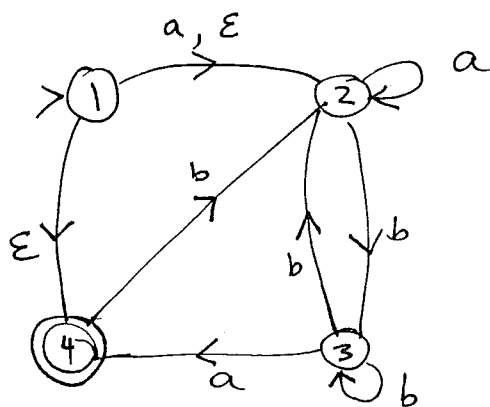
$$a(\epsilon + a^*b)^3 a^*$$

$$+ b(\epsilon + a^*b)^2 a^* + b(\epsilon + a^*b)a^* + a^*(b + \epsilon)a^*(b + \epsilon)a^*(b + \epsilon)a^*$$

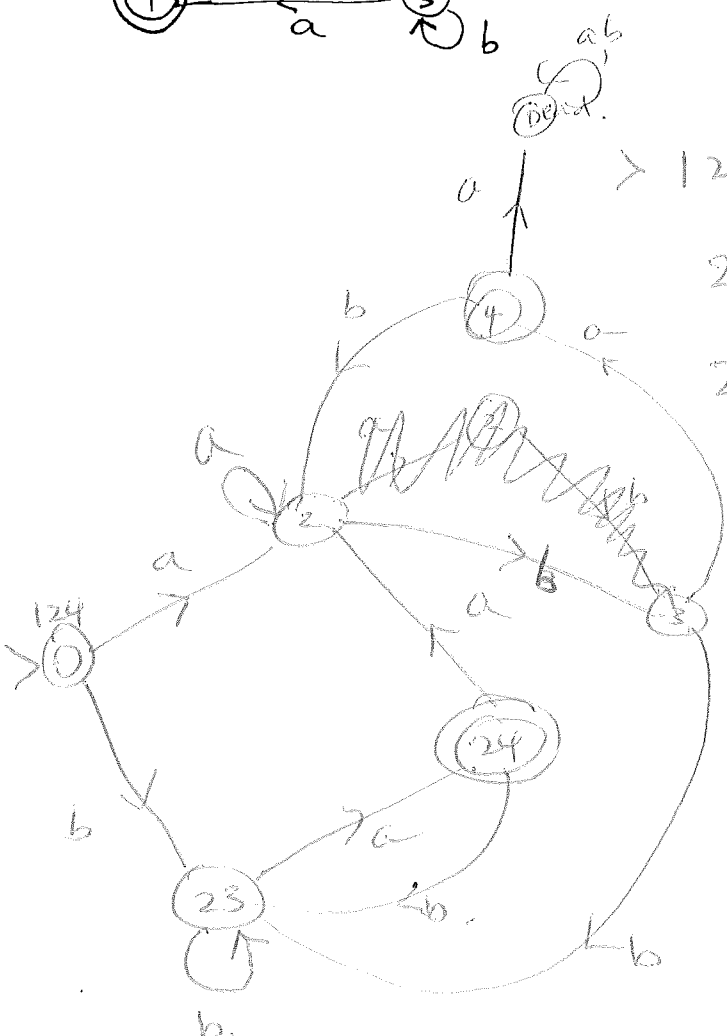
6. (3 marks) Give a regular expression for the language of strings over $\{a, b\}$ that have an odd number of a 's and, if it starts with an a , then it ends in b .

$$a(b^*ab^*ab^*)^*b + b^+a^+b^+ \\ b(b^*ab^*ab^*)^*ab^*$$

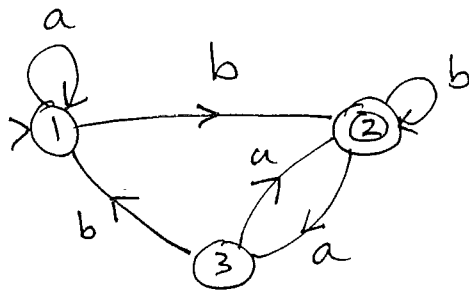
7. (5 marks) Find, using the construction, a DFA that is equivalent to the following NFA. Show the dead state and all transitions to it.



	a	b
1	2	∅
2	2	3
3	∅ , 4	2, 3
4	∅	2
∅	∅	∅



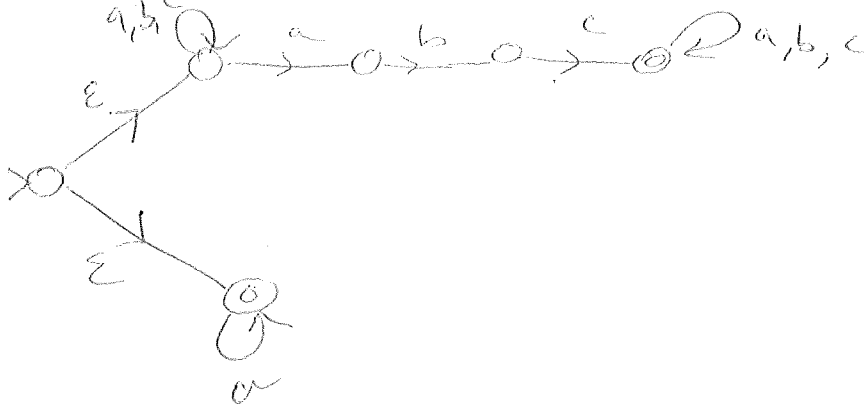
8. (5 marks) Use the construction to find the RE that describes the language accepted by the following FA.



$$a^*b (aba^*b + aa)^* + b)^*$$

9. (4 marks) Draw a Non-deterministic finite automaton that accepts the following language:

$\{w \in \{a, b, c\}^* : w \text{ contains } abc \text{ as a substring or } \#_b(w) = 0\}$

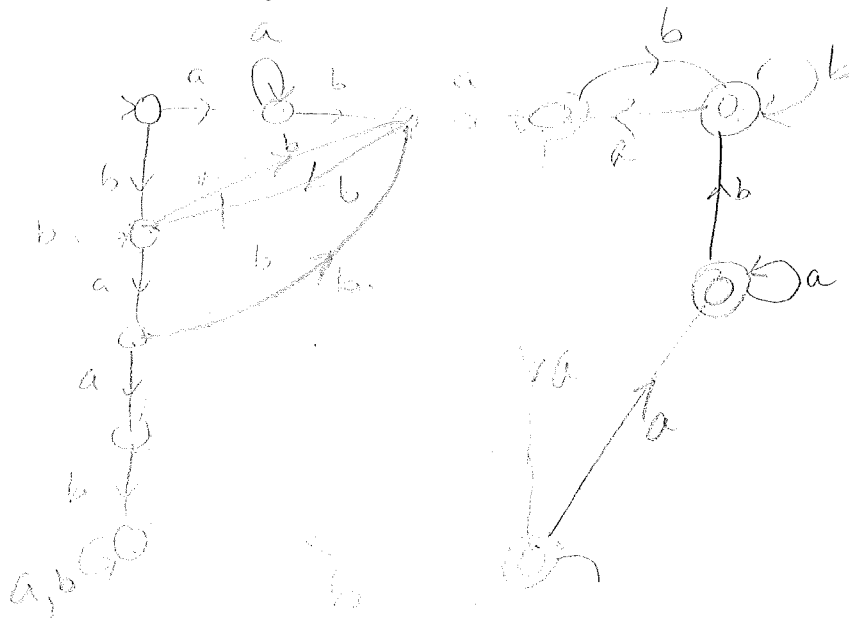


10. (2 marks) Give a regular expression for the language

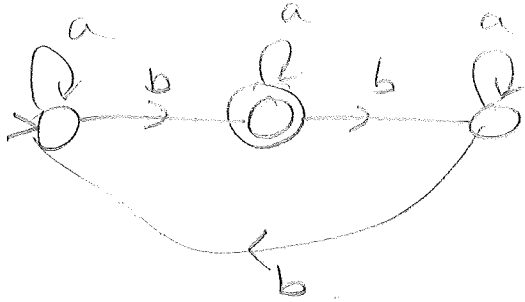
$\{w \in \{a, b\}^* : w \text{ begins and ends in the same letter and has at least 3 a's}\}$

$$a^*(b^*ab^*)^2(a^*b^*)^*a + b^*(b^*ab^*)^2(a^*b^*)^*b + a b^* a (b+a)^* a$$

11. (4 marks) Give a DFA for the language $L = \{w \in \{a, b\}^* : w \text{ does not contain } baab \text{ but it does contain } aba\}$.



12. (2 marks) Give a DFA for the language $L = \{w \in \{a,b\}^* : w \text{ has a number of } b\text{'s that is congruent to 1 modulo 3}\}$.



13. (5 marks) Show that the set of ordered pairs of integers is countable. Examples of elements of this set are $(-2, 11)$, $(11, -2)$, $(0,0)$, $(14, 1)$, $(-3,-3)$, etc.

